Decorrelation of vector fields with speed of varifolds

Infinite-Dimensional Geometry : Theory and Applications

Rayane Mouhli

Joint work with Barbara Gris and Irène Kaltenmark

Université Paris Cité (MAP5) & Sorbonne Université (LJLL)

LDDMM (Beg, Miller, Trouvé, Younes 2005)

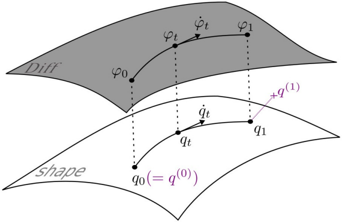
Let $v \in L^2([0,1], V)$ be a time-varying vector field where $V \hookrightarrow \mathcal{C}_0^2(\mathbb{R}^d, \mathbb{R}^d)$.
The flow of diffeomorphism φ^v generated by v is the unique solution of :

$$\dot{\varphi}_t^v = v_t \circ \varphi_t^v \qquad \text{s.t} \quad \varphi_0^v = \mathrm{id}$$

Shape registration corresponds to the following energy minimization problem :

$$\min_{v \in L^2([0,1],V)} E(v) = \int_0^1 \frac{1}{2}|v_t|_V^2 \, dt + D(\varphi_1 \cdot q^{(0)}, q^{(1)})$$

$$\text{s.t} \quad \dot{q}_t = v_t \cdot q_t \text{ and } q_0 = q^{(0)}$$



Coupling two types of deformations

Let $v \in L^2([0,1], V), w \in L^2([0,1], W)$ be two vector fields and ψ its associated diffeomorphism.

$$\dot{\psi}_t = (v_t + w_t) \cdot \psi_t \qquad \text{s.t} \qquad \psi_0 = \mathrm{id}$$

Given a source $q^{(0)}$, the deformed shape $q_t = \psi_t(q^{(0)})$ follows the dynamic

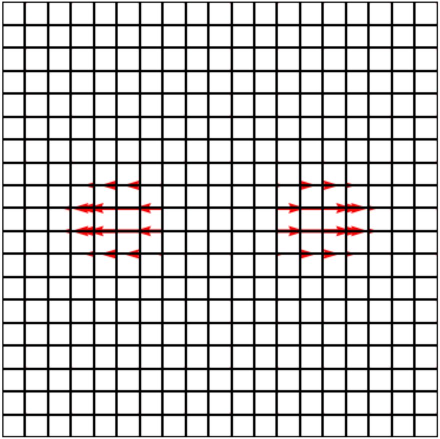
$$\dot{q}_t = v_t \cdot q_t + w_t \cdot q_t$$

The energy minimization problem associated is

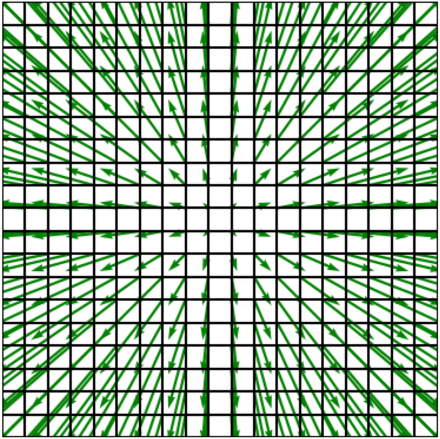
$$\min_{v,w \in L^2([0,1], V \times W)} E(v,w) = \int_0^1 \frac{1}{2}|v_t|_V^2 + \frac{1}{2}|w_t|_W^2 dt + \mathcal{A}(q_1)$$

where $\mathcal{A} : Q \to \mathbb{R}$ is a data attachment term

Decorrelation with respect to a shape

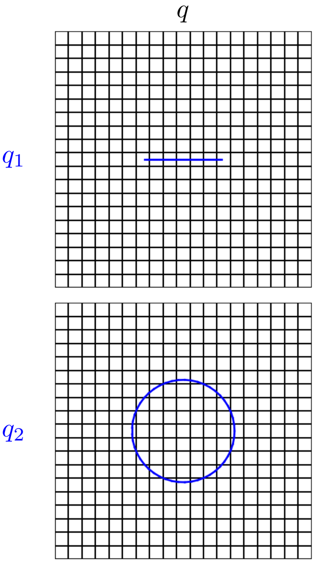


v_1



v_2

Decorrelation with respect to a shape



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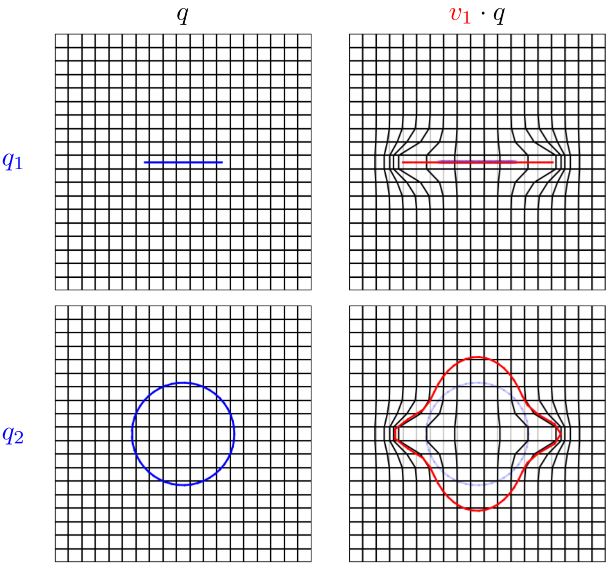
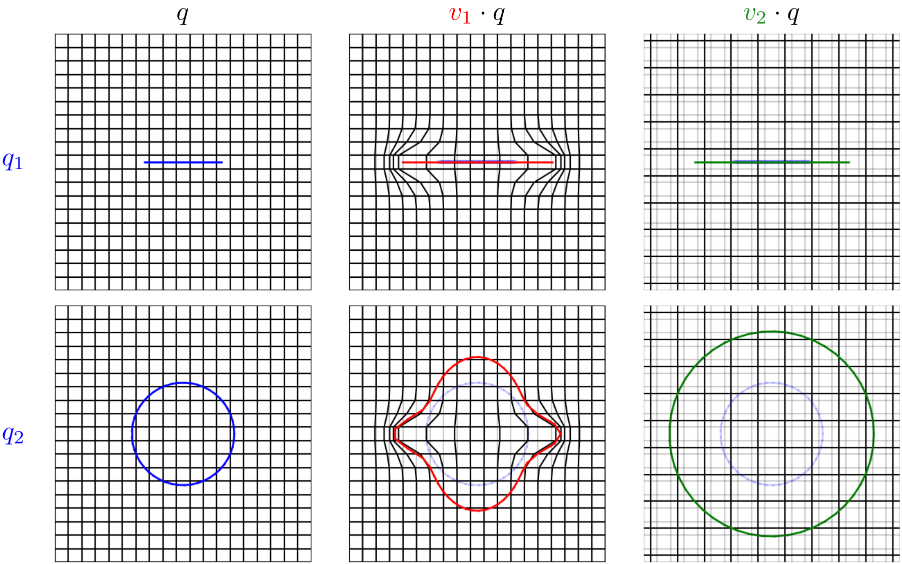


Decorrelation with respect to a shape



Correlation with respect to a shape

We define the correlation with respect to a shape q between a vector field $v \in V$ and a space of vector fields W by

$$\mathrm{Corr}_q(v, W) = \|w^*\|_W$$

where

$$w^* = \underset{w \in W}{\mathrm{argmin}} \, \|\delta\mu_q(v) - \delta\mu_q(w)\|_{\mathcal{W}}^2 + \lambda \|w\|_W^2$$

and $\mathcal{W} \hookrightarrow C_0^1(\mathbb{R}^d, \mathbb{R}^d)$ is a Reproducing Kernel Hilbert Space

Varifold

Definition

A varifold is a continuous linear form on $\Omega = \{\omega : \mathbb{R}^d \times \mathbb{S}^{d-1} \to \mathbb{R}\}$.
The varifold μ_q associated to the shape $q : X \to \mathbb{R}^d$ is defined by :

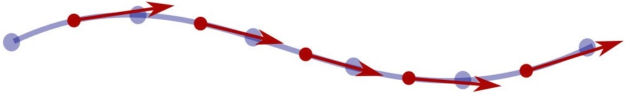
$$\mu_q(\omega) = \int_X \omega(x, \vec{t}(x))\, dx$$

where $\vec{t}$ represents a tangent/normal vector to the curve/surface.

A discrete curve can be modeled by a varifold

$$\mu_q(\omega) = \sum_{(f^1, f^2) \in F} \|q_{f^2} - q_{f^1}\| \, \omega(c(q_f), \vec{t}(q_f))$$

where $c(q_f) = \frac{q_{f^1} + q_{f^2}}{2}$ and $\vec{t}(q_f) = \frac{q_{f^2} - q_{f^1}}{\|q_{f^2} - q_{f^1}\|}$.



Properties

Proposition

Given a RKHS $\mathcal{W} \hookrightarrow C_0^0(\mathbb{R}^d \times \mathbb{S}^{d-1})$ generated by a kernel $k_{\mathcal{W}} = k_E \otimes k_T$ and two curves q_a and q_b represented by $\mu_{q_a}, \mu_{q_b} \in W'$, there exists a scalar product $\langle \mu_{q_a}, \mu_{q_b} \rangle$.

Proposition

The action of a diffeomorphism on a varifold is defined by

$$(\phi_* \mu_q)(\omega) = \mu_{\phi(q)}(\omega) = \sum_{(f^1,f^2)\in F} \|\phi(q_{f^2}) - \phi(q_{f^1})\| \, \omega\left(c(\phi(q_f)), \vec{t}(\phi(q_f))\right)$$

Speed of a varifold induced by a vector field

Theorem (Charon, Trouvé 2013)

Let $t \mapsto \phi_t$ *be a flow of diffeomorphism such that* $\phi_0 = \mathrm{id}$ *and* $\dot{\phi}_t|_{t=0} = v$

$$\begin{aligned} \frac{d}{dt}\Big|_{t=0} \mu_{\phi_t(q)}(\omega) &= \sum_{(f^1,f^2)\in F} \frac{\langle v(q_{f^2}) - v(q_{f^1}), q_{f^1} - q_{f^2}\rangle}{\|q_{f^1} - q_{f^2}\|}\, \omega(c(q_f), \vec{t}(q_f)) \\ &+ \|q_{f^1} - q_{f^2}\| \left(\partial_x \omega(c(q_f), \vec{t}(q_f)) \Big| v(c(q_f))\right) \\ &+ \|q_{f^1} - q_{f^2}\| \left(\partial_{\vec{t}} \omega(c(q_f), \vec{t}(q_f)) \Big| \nabla^\perp v(q_f)\right) \end{aligned}$$

where $\nabla^\perp v(q_f) = \frac{v(q_{f^2}) - v(q_{f^1})}{\|q_{f^1} - q_{f^2}\|} - \left(\frac{v(q_{f^2}) - v(q_{f^1})}{\|q_{f^1} - q_{f^2}\|} \cdot \vec{t}(q_f)\right) \vec{t}(q_f)$

In the following, we will denote $\frac{d}{dt}\Big|_{t=0} \mu_{\phi_t(q)}$ as $\delta\mu_q(v)$

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&+ \|q_{f^1} - q_{f^2}\| \left(\partial_{\vec{t}} \omega(c(q_f), \vec{t}(q_f)) \Big| \nabla^\perp v(q_f)\right)
\end{aligned}$$

where $\nabla^\perp v(q_f) = \frac{v(q_{f^2}) - v(q_{f^1})}{\|q_{f^1} - q_{f^2}\|} - \left(\frac{v(q_{f^2}) - v(q_{f^1})}{\|q_{f^1} - q_{f^2}\|} \cdot \vec{t}(q_f)\right) \vec{t}(q_f)$

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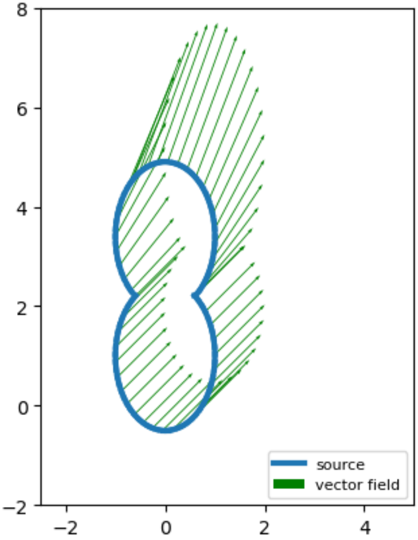
Influence of σ : $k(r) = e^{-r^2/\sigma^2}$



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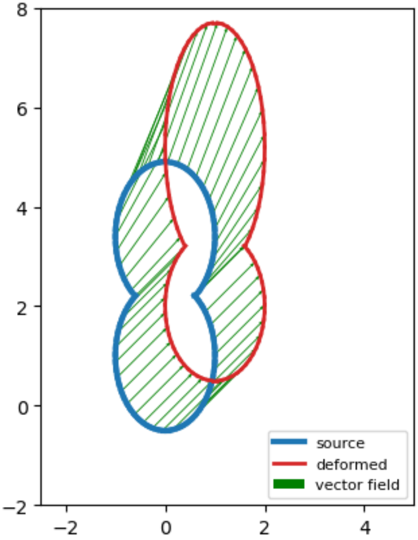

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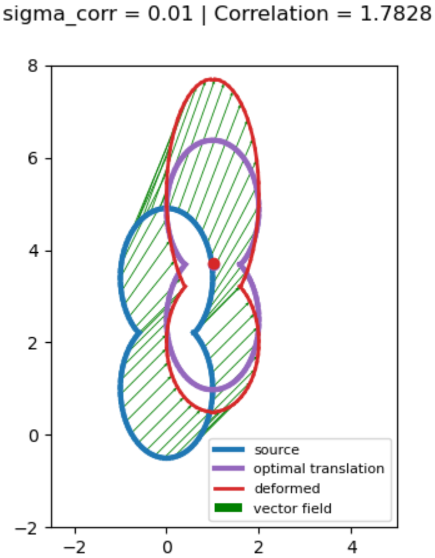

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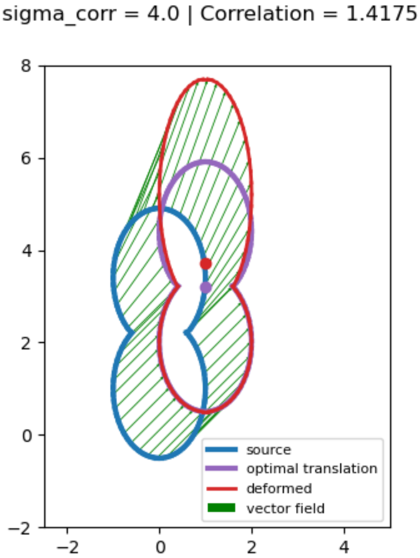

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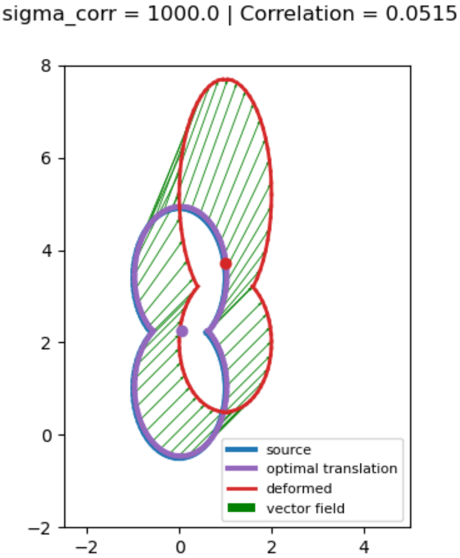

Dynamic generated by two vector fields

Given V, W two spaces of vector fields, we are interested in the following energy-minimization problem

$$\min_{(v,w)\in L^2([0,1],V\times W)} E(v,w) = \int_0^1 \frac{1}{2}|v_t|_V^2 + \frac{1}{2}|w_t|_W^2 dt + \mathcal{A}(q_1)$$
$$\text{s.t} \quad \dot{q}_t = v_t \cdot q_t + w_t \cdot q_t$$

where $\mathcal{A} : \mathcal{Q} \to \mathbb{R}$ is a data attachment term.

Different approaches in the litterature :

- Multiscale kernel bundle, sum of gaussian kernel : Sommer et al. 2013, Risser 2011
- Semidirect product : Bruveris et al. (2010, 2012)
- Hierarchical model : Pierron et al. 2024

Direct model

We consider the flow of diffeomorphism ψ generated by $(v, w) \in L^2([0,1], V \times W)$

$$\dot{\psi}_t = (v_t + w_t) \circ \psi_t \quad \text{where} \quad \psi_0 = \mathrm{id}$$

Considering the deformed shape $q_t = \psi_t(q^{(0)})$ the energy minimization problem is equivalent to

$$\min_{p_0} E(p_0) = \int_0^1 \frac{1}{2}|v_t|_V^2 + \frac{1}{2}|w_t|_W^2 dt + \mathcal{A}(q_1)$$

$$\text{s.t} \quad \begin{cases} \dot{q}_t & = \xi_{q_t}^V(v_t) + \xi_{q_t}^W(w_t) \\ \dot{p}_t & = -(\partial_q \xi_{q_t}^V(v_t) + \partial_q \xi_{q_t}^W(w_t))^* p_t \\ v_t & = K_V \xi_{q_t}^{V*} p_t \\ w_t & = K_W \xi_{q_t}^{W*} p_t \end{cases}$$

where $\xi_{q_t}^V(v_t) = v_t \cdot q_t$, $\xi_{q_t}^W(w_t) = w_t \cdot q_t$ and $p_t \in T_{q_t}^* Q$

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Direct model

We penalize the energy with the correlation to define a new problem.

$$\min_{p_0} E(p_0) = \int_0^1 \frac{1}{2}|v_t|_V^2 + \frac{1}{2}|w_t|_W^2 dt + \gamma \int_0^1 \frac{1}{2} \mathrm{Corr}_{q_t}(v_t, W)^2 dt + \mathcal{A}(q_1)$$

$$\text{s.t} \quad \begin{cases} \dot{q}_t &= \xi_{q_t}^V(v_t) + \xi_{q_t}^W(w_t) \\ \dot{p}_t &= -(\partial_q \xi_{q_t}^V(v_t) + \partial_q \xi_{q_t}^W(w_t))^* p_t \\ v_t &= K_V \xi_{q_t}^{V*} p_t \\ w_t &= K_W \xi_{q_t}^{W*} p_t \end{cases}$$

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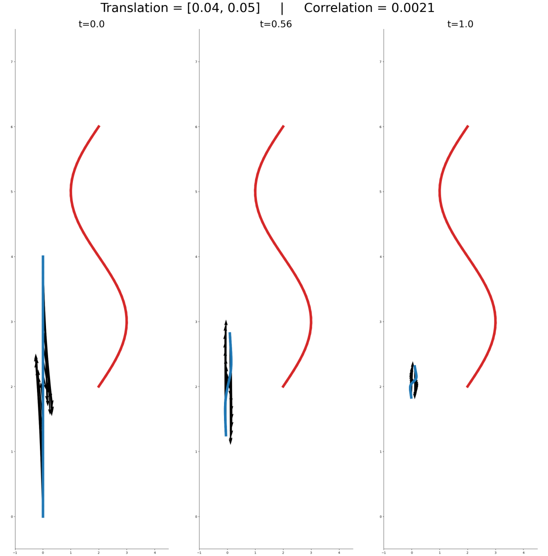
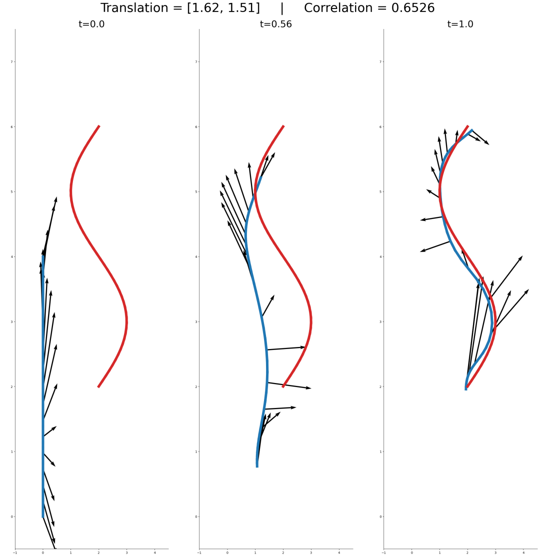
Decorrelation with one moment



Decorrelation with one moment



Direct model

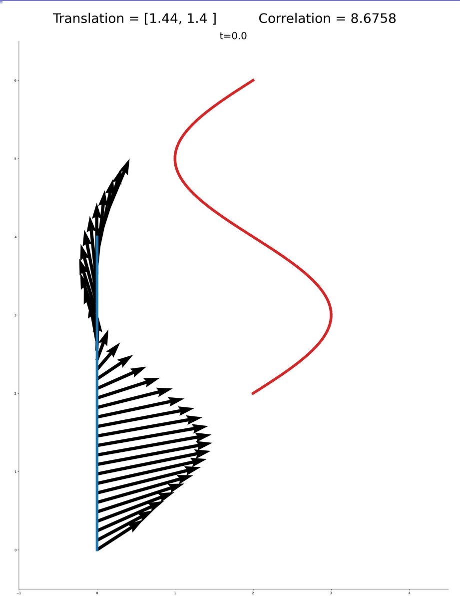
Considering the partial gradient of $E(v, w)$, we can define a new problem parameterized by two moments $(p_t^V, p_t^W) \in T^*_{q_t}Q \times T^*_{q_t}Q$.

$$\min_{p_0^V, p_0^W} E(p_0^V, p_0^W) = \int_0^1 \frac{1}{2}|v_t|_V^2 + \frac{1}{2}|w_t|_W^2 dt + \lambda \int_0^1 \frac{1}{2} \operatorname{Corr}_{q_t}(v_t, W)^2 dt + \mathcal{A}(q_1)$$

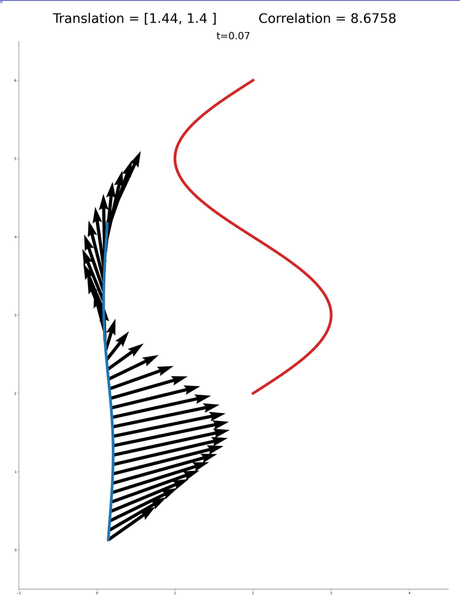
$$\text{s.t} \quad \begin{cases} \dot{q}_t &= v_t \cdot q_t + w_t \cdot q_t \\ \dot{p}_t^V &= -(\partial_q \xi_{q_t}^V(v_t) + \partial_q \xi_{q_t}^W(w_t))^* p_t^V \\ \dot{p}_t^W &= -(\partial_q \xi_{q_t}^V(v_t) + \partial_q \xi_{q_t}^W(w_t))^* p_t^W \\ v_t &= K_V \xi_{q_t}^{V*} p_t^V \\ w_t &= K_W \xi_{q_t}^{W*} p_t^W \end{cases}$$

where $\xi_{q_t}^V(v_t) = v_t \cdot q_t$, $\xi_{q_t}^W(w_t) = w_t \cdot q_t$.

Direct model without decorrelation



Direct model without decorrelation



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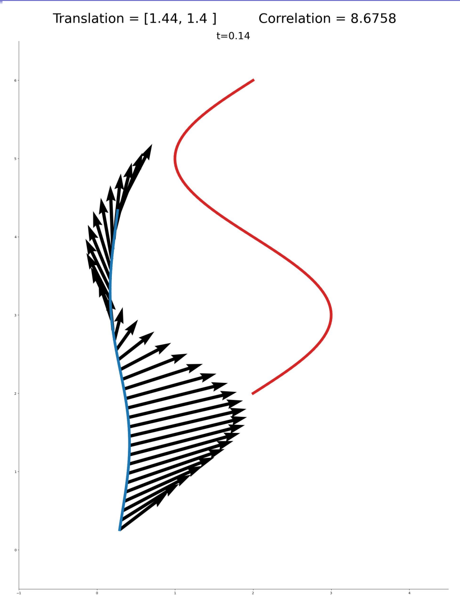

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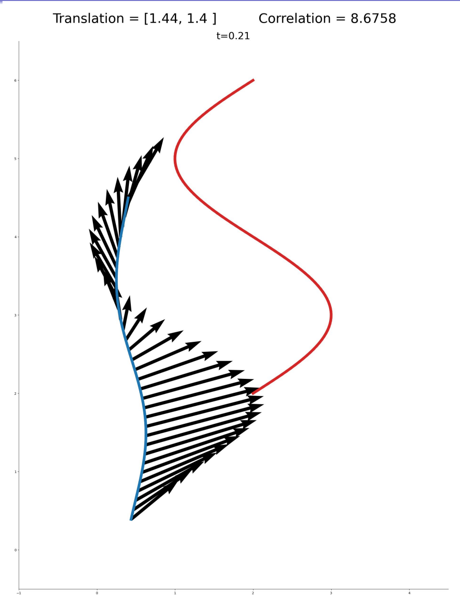
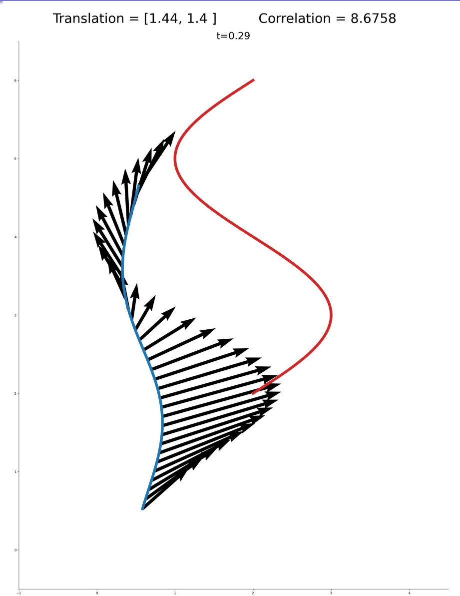


Direct model without decorrelation



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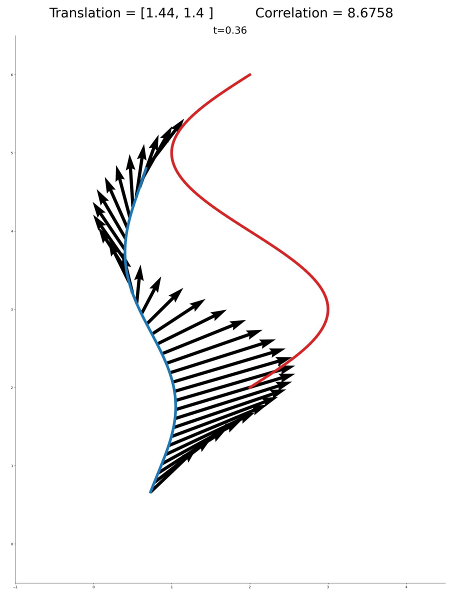

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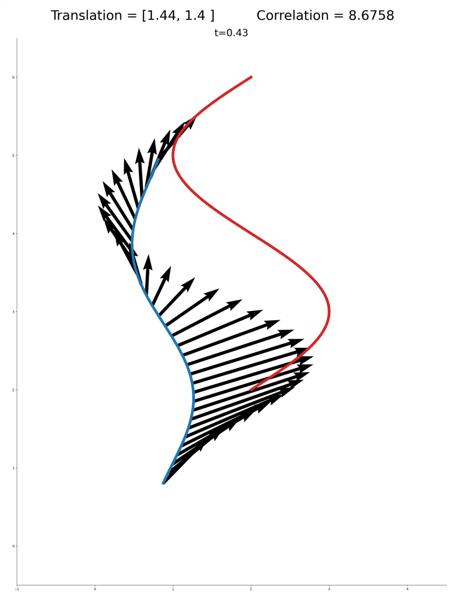

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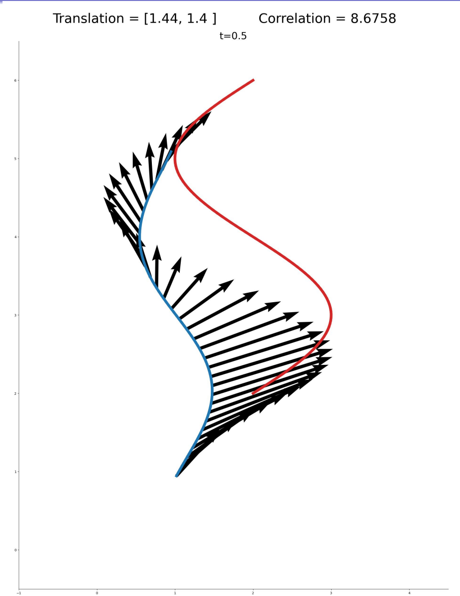

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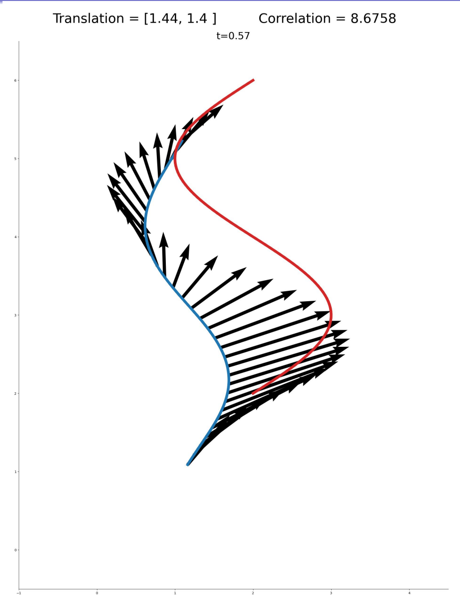

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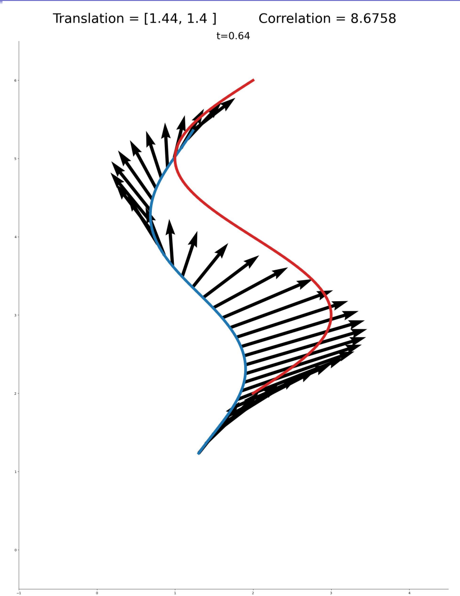
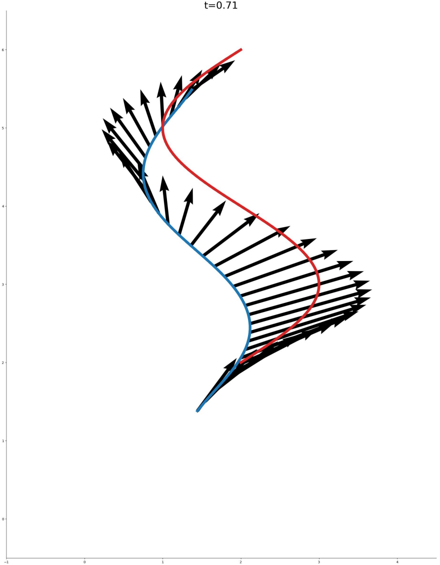


Direct model without decorrelation

Translation = [1.44, 1.4] Correlation = 8.6758



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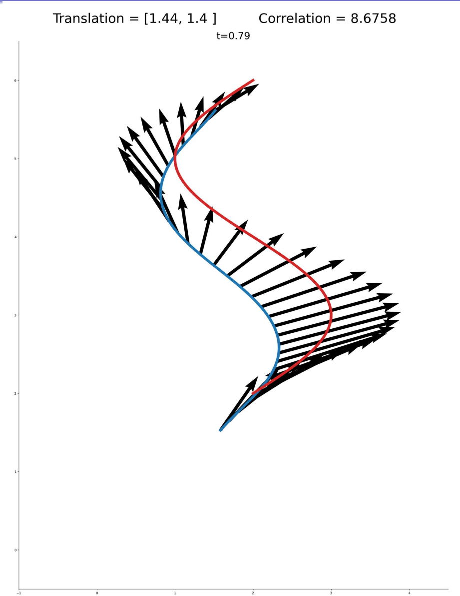

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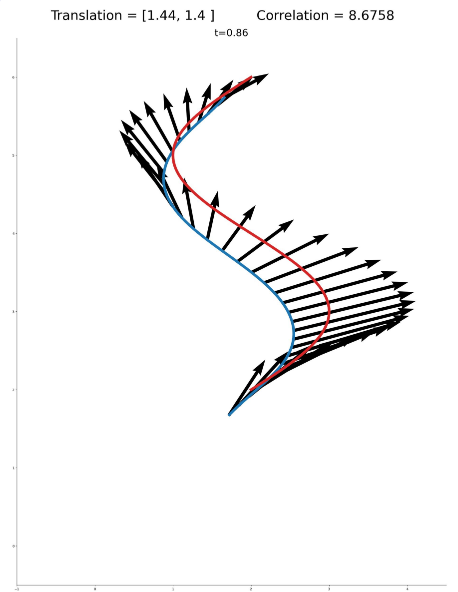

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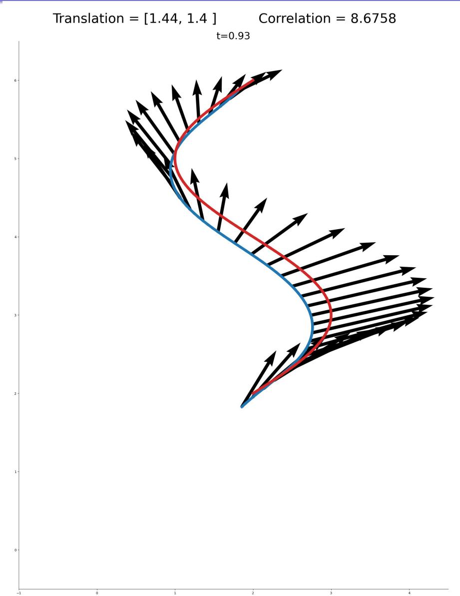
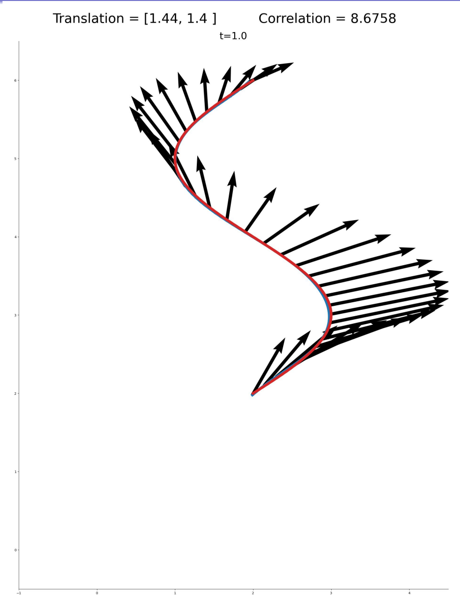
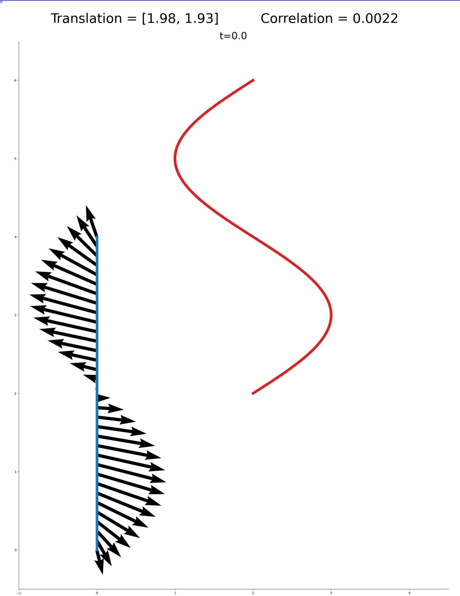


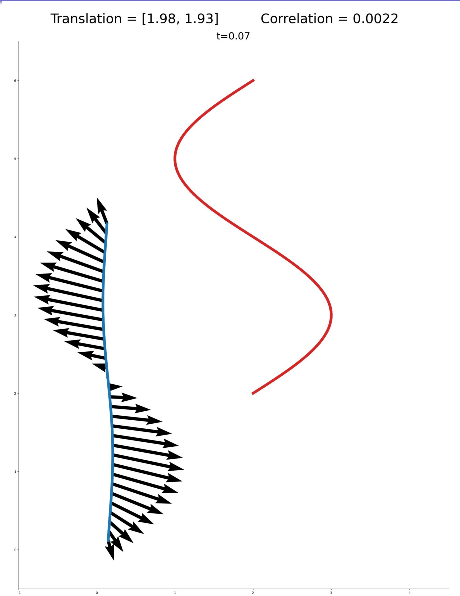
Direct model without decorrelation



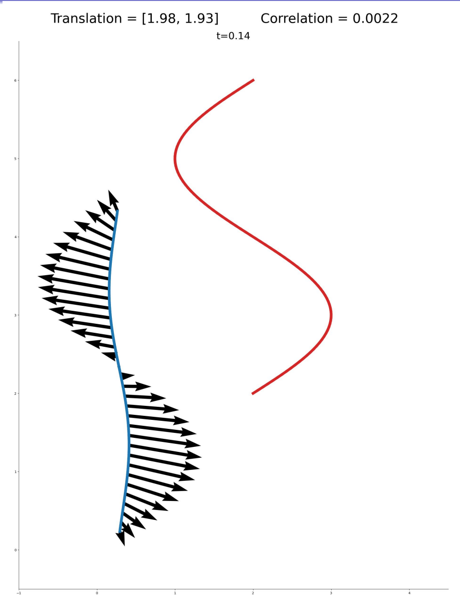
Direct model with decorrelation



Direct model with decorrelation



Direct model with decorrelation



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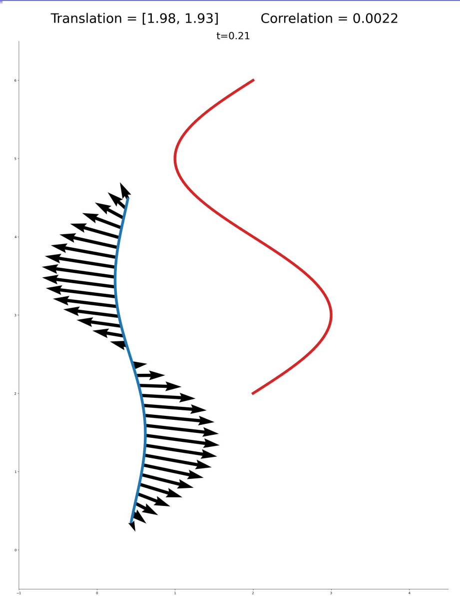
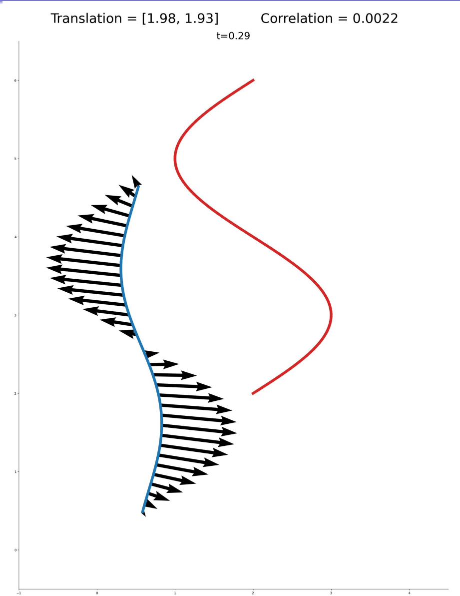
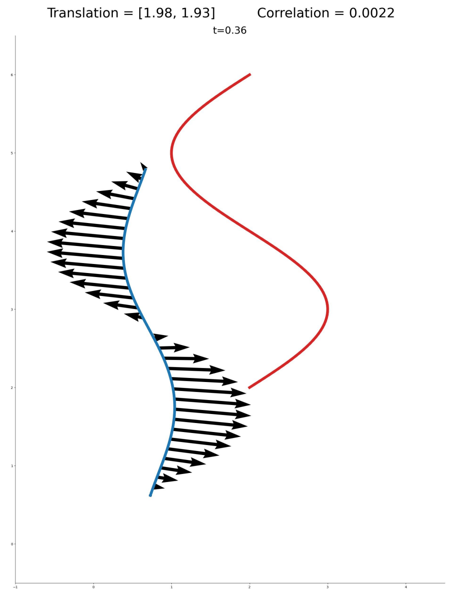


Direct model with decorrelation



Direct model with decorrelation



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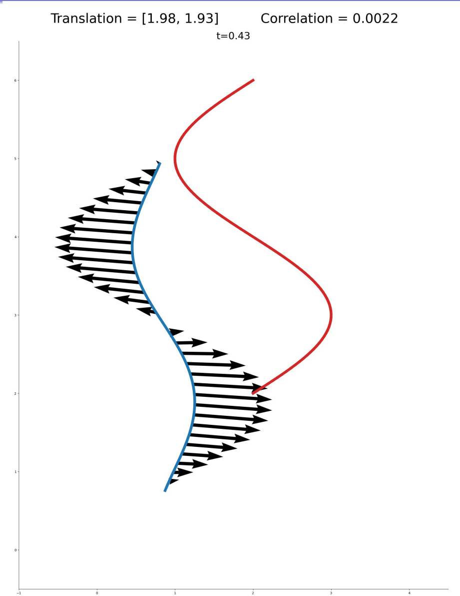

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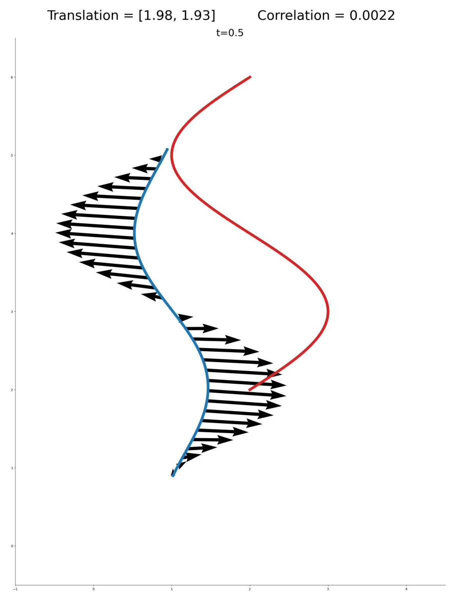

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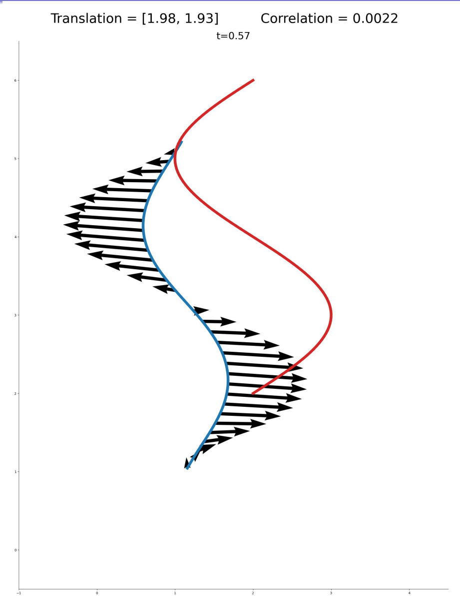

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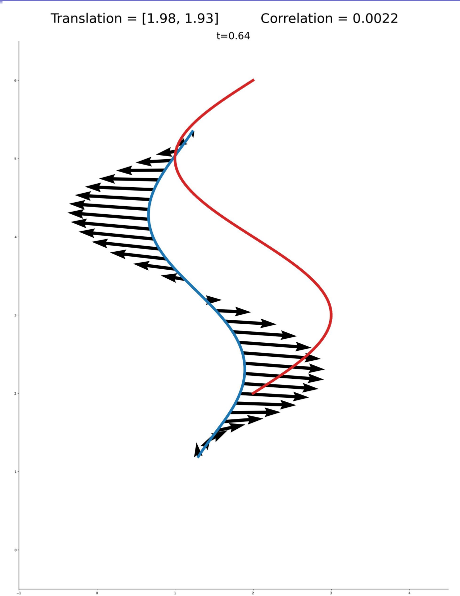

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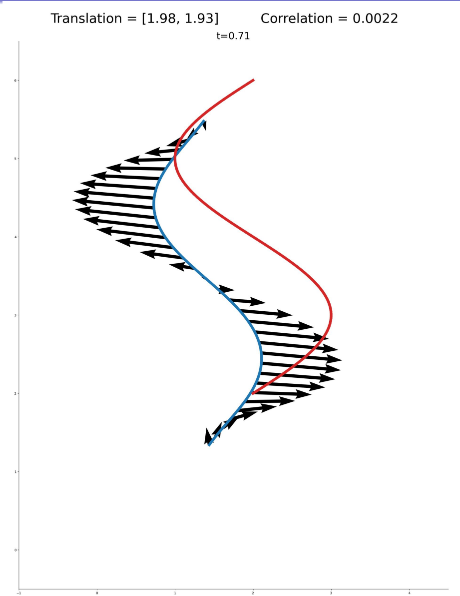

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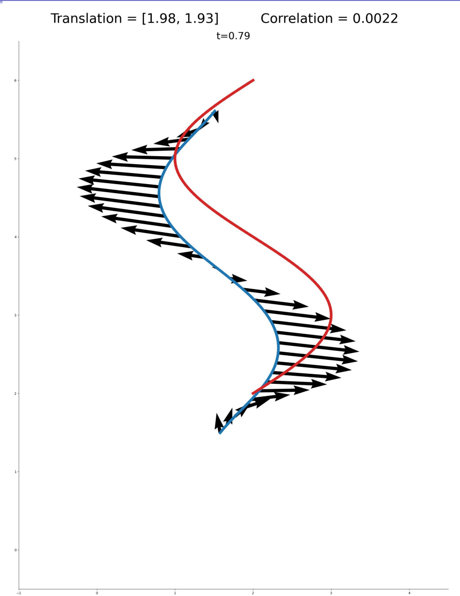

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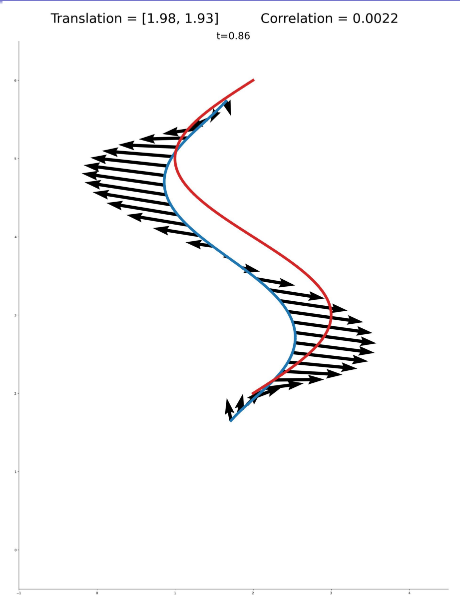

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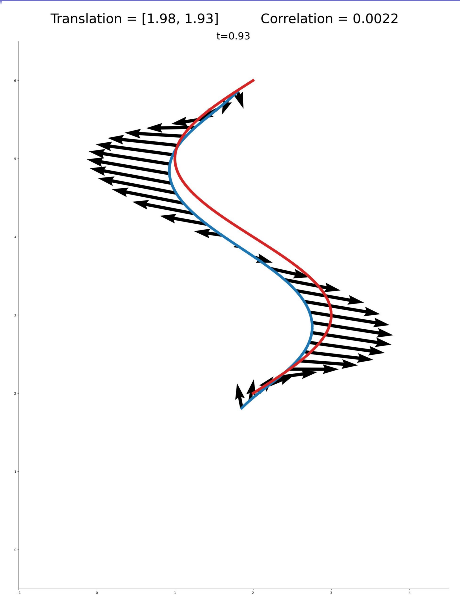

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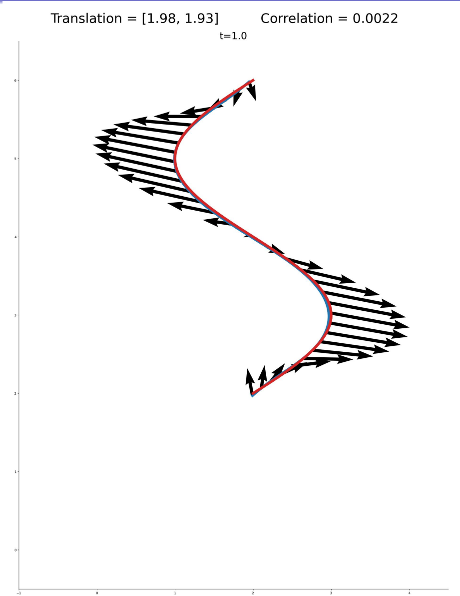


Semidirect model (joint work with Thomas Pierron)

Let G be a finite dimensional Lie group and $\mathfrak{g}$ its Lie algebra. We denote $\alpha_g(\varphi)$ the action of G on $\mathrm{Diff}_{C_0^k}(\mathbb{R}^d)$.

We consider the semidirect product $\mathcal{G} = G \ltimes \mathrm{Diff}_{C_0^k}(\mathbb{R}^d)$ and we assume it acts on Q as follow :

$$(g, \varphi) \cdot q = g \cdot (\varphi \cdot q)$$

Example : If $G = \mathbb{R}^d$, then $\alpha_T(\varphi)(x) = \varphi(x + T) - T$ and $(T, \varphi) \cdot q = \varphi(q) + T$.

Semidirect model

The minimization problem associated to the semidirect model is

$$\min_{p_0} E(p_0) = \int_0^1 \frac{1}{2}|v_t|_V^2 + \frac{1}{2}|X_t|_{\mathfrak{g}}^2 dt + \mathcal{A}(q_1)$$

$$\text{s.t} \quad \begin{cases} \dot{q}_t & = v_t \cdot q_t + X_t \cdot q_t \\ \dot{p}_t & = -(\partial_q \xi_{q_t}^V(v_t) + \partial_q \xi_{q_t}^{\mathfrak{g}}(X_t))^* p_t \\ v_t & = K_V \xi_{q_t}^{V*} p_t \\ X_t & = K_{\mathfrak{g}} \xi_{q_t}^{\mathfrak{g}*} p_t \end{cases}$$

where $\xi_{q_t}^{\mathfrak{g}}(X_t) = X_t \cdot q_t$ and $p_t \in T_{q_t}^* Q$

Semidirect model

The minimization problem associated to the semidirect model is

$$\min_{p_0} E(p_0) = \int_0^1 \frac{1}{2}|v_t|_V^2 + \frac{1}{2}|X_t|_{\mathfrak{g}}^2 dt + \mathcal{A}(q_1)$$

$$\text{s.t} \quad \begin{cases} \dot{q}_t & = v_t \cdot q_t + X_t \cdot q_t \\ \dot{p}_t & = -(\partial_q \xi_{q_t}^V(v_t) + \partial_q \xi_{q_t}^{\mathfrak{g}}(X_t))^* p_t \\ v_t & = K_V \xi_{q_t}^{V*} p_t \\ X_t & = K_{\mathfrak{g}} \xi_{q_t}^{\mathfrak{g}*} p_t \end{cases}$$

where $\xi_{q_t}^{\mathfrak{g}}(X_t) = X_t \cdot q_t$ and $p_t \in T_{q_t}^* Q$

Semidirect model

We define a new shape $\tilde{q} = g^{-1} \cdot q$, in particular the deformation of $\tilde{q}$ is

$$\tilde{q}_t = \varphi_t \cdot q^{(0)}$$

Considering the augmented shape space $G \times Q$ and a new data attachment term

$$\tilde{A}(g, \tilde{q}) = \mathcal{A}(g \cdot \tilde{q}) = \mathcal{A}(q)$$

will allow us to consider two moments $p^{\mathfrak{g}} \in T^*G$ and $\tilde{p} \in T^*Q$.

Semidirect model

The minimization energy problem can be written with two moments $(\tilde{p}_t, p_t^{\mathfrak{g}}) \in T^*_{\tilde{q}_t} Q \times T^*_{g_t} G$:

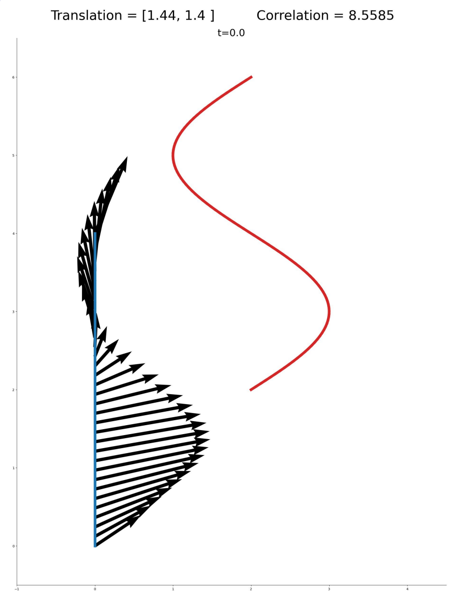
$$\min_{\tilde{p}_0, p_0^{\mathfrak{g}}} E(\tilde{p}_0, p_0^{\mathfrak{g}}) = \int_0^1 \frac{1}{2}|v_t|_V^2 + \frac{1}{2}|X_t|_{\mathfrak{g}}^2 dt + \lambda \int_0^1 \frac{1}{2} \mathrm{Corr}_{q_t}(v_t, \mathfrak{g})^2 dt + \tilde{\mathcal{A}}(g_1, \tilde{q}_1)$$

s.t

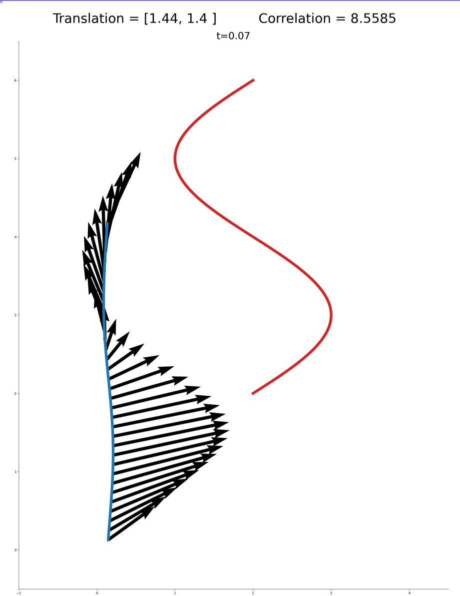
$$\begin{cases} \dot{g}_t & = X_t \cdot g_t \\ \dot{\tilde{q}}_t & = d_{\mathrm{id}} \alpha_{g_t}(v_t) \cdot \tilde{q}_t \\ \dot{\tilde{p}}_t & = -(\partial_q \xi^{\tilde{V}}_{\tilde{q}_t}(d_{\mathrm{id}} \alpha_{g_t}(v_t))^* \tilde{p}_t \\ \dot{p}_t^{\mathfrak{g}} & = -(\partial_g \xi^{\mathfrak{g}}_{g_t}(X_t))^* p_t^{\mathfrak{g}} - (\partial_g \xi^V_{\tilde{q}_t}(d_{\mathrm{id}} \alpha_{g_t}(v_t)))^* \tilde{p}_t \\ v_t & = K_V (\xi^V_{\tilde{q}_t} d_{\mathrm{id}} \alpha_{g_t})^* \tilde{p}_t \\ X_t & = K_{\mathfrak{g}} \xi^{\mathfrak{g}*}_{g_t} p_t^{\mathfrak{g}} \end{cases}$$

where $\xi^{\mathfrak{g}}_{g_t}(X_t) = X_t g_t$

Semidirect model without decorrelation



Semidirect model without decorrelation



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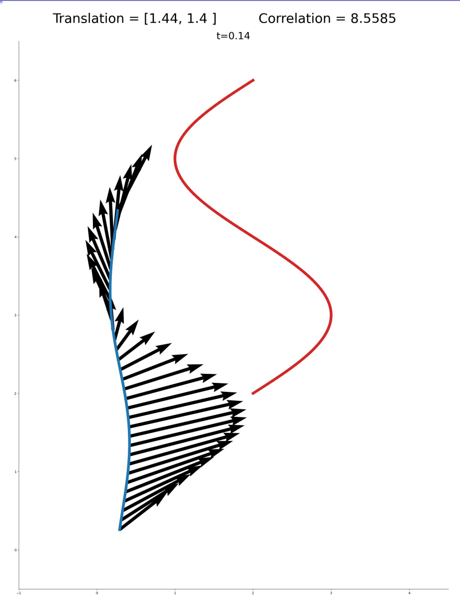

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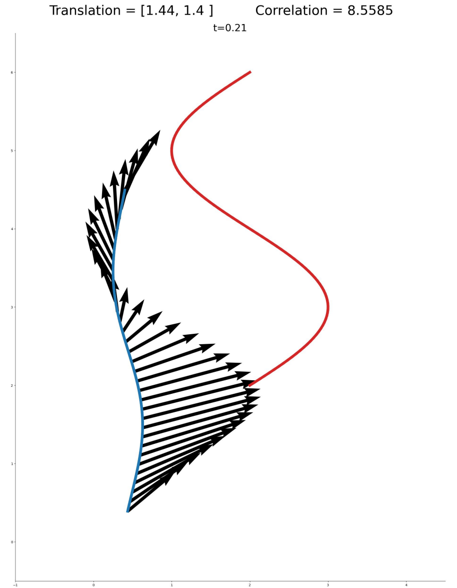

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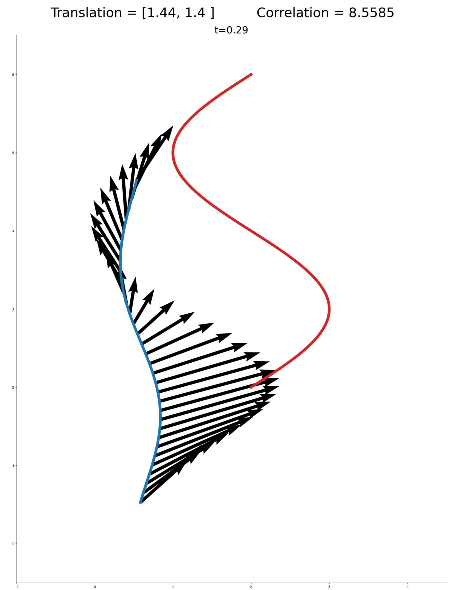

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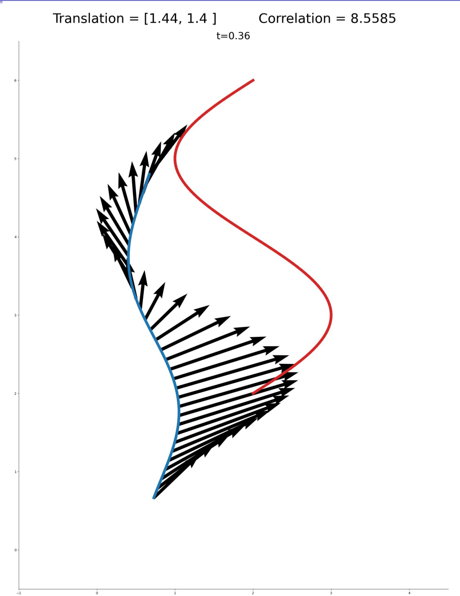

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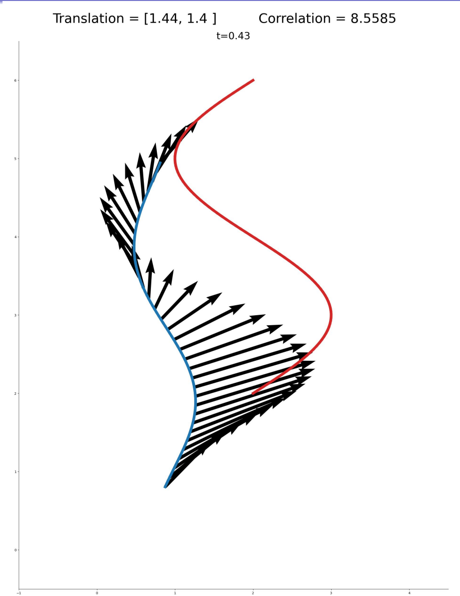

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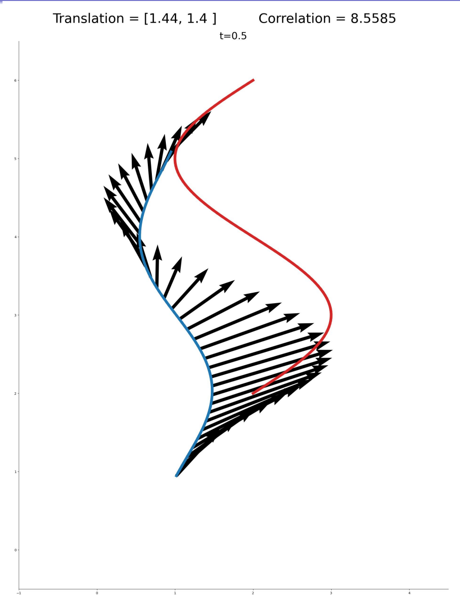

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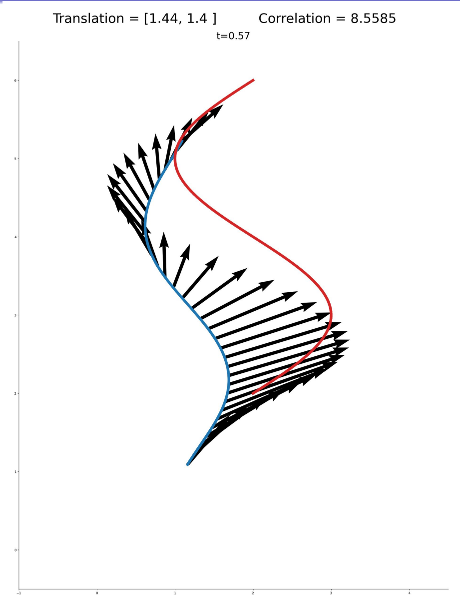

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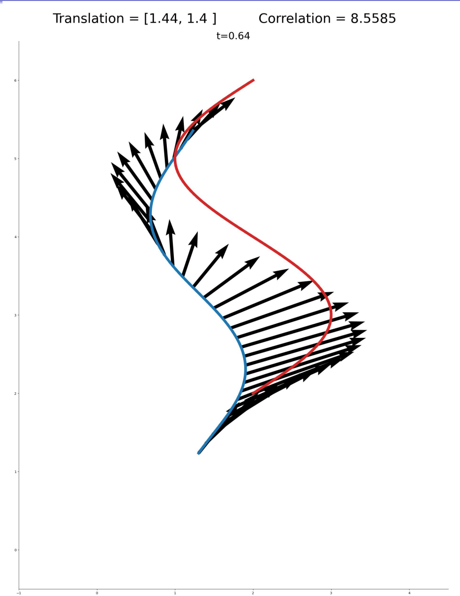

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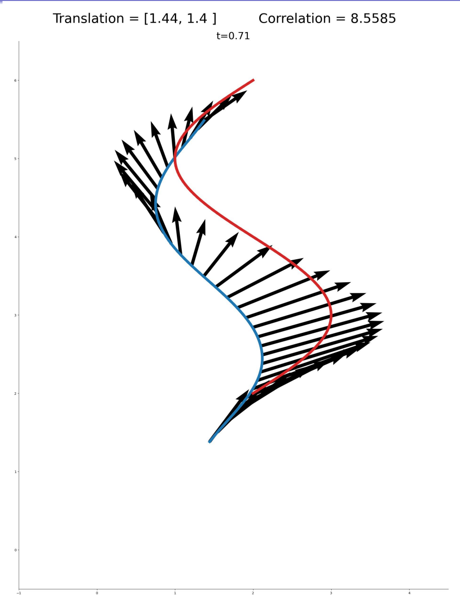

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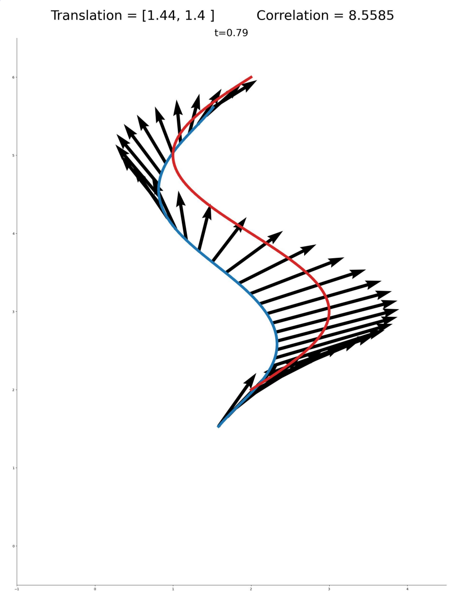

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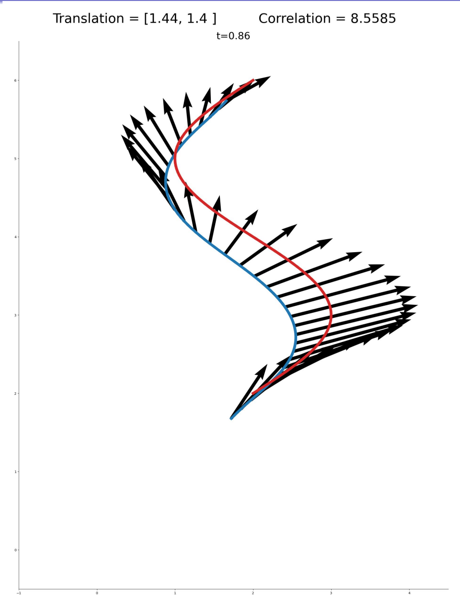

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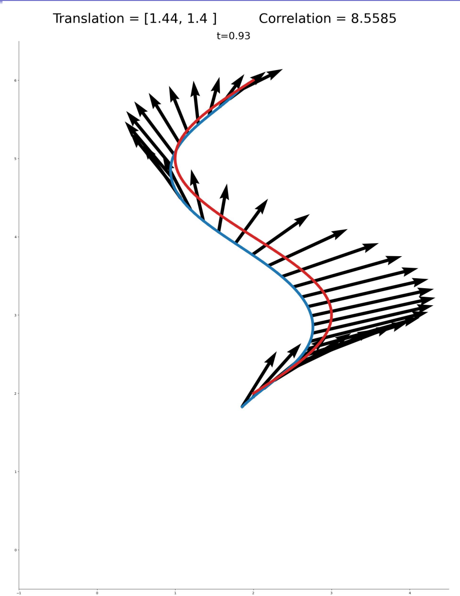

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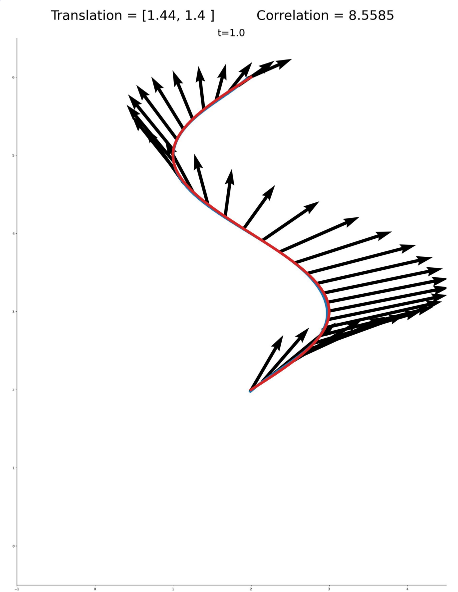

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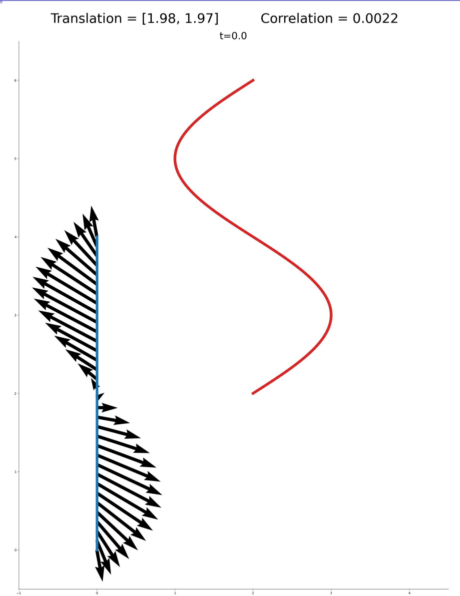

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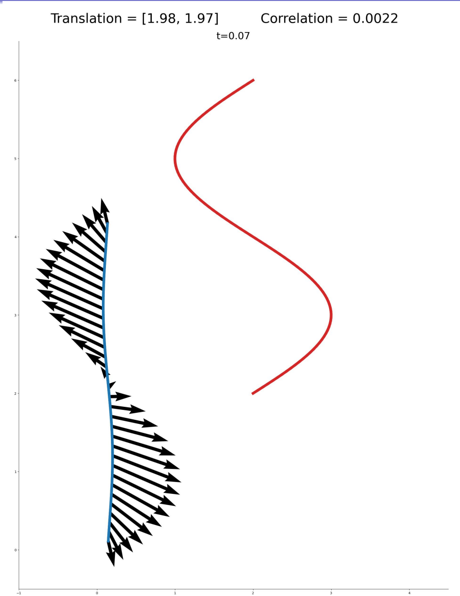

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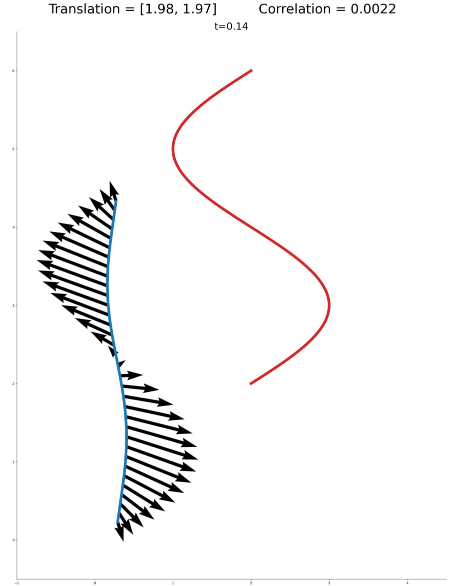

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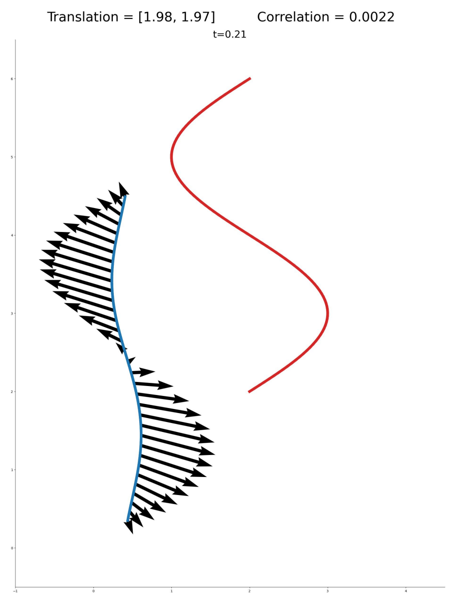

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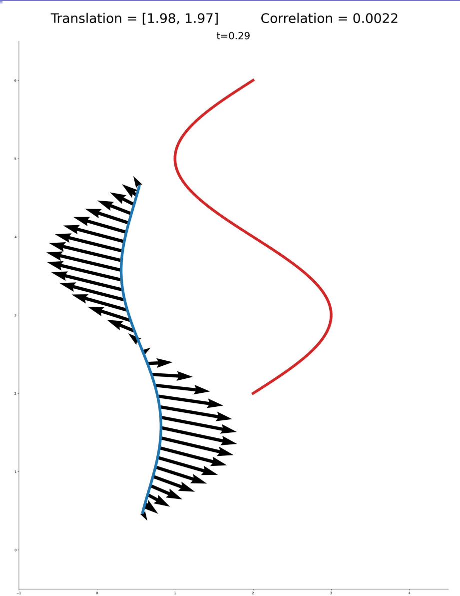

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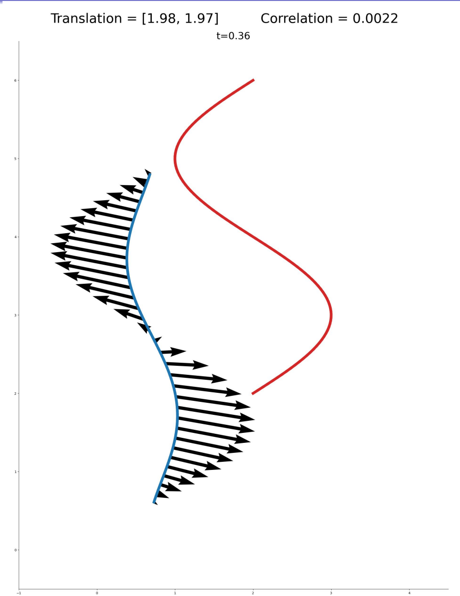

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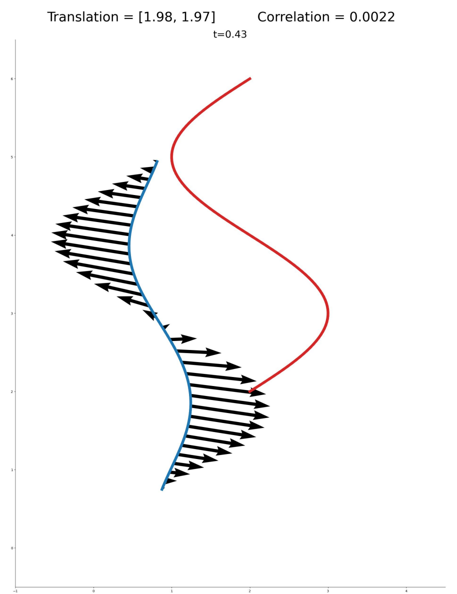

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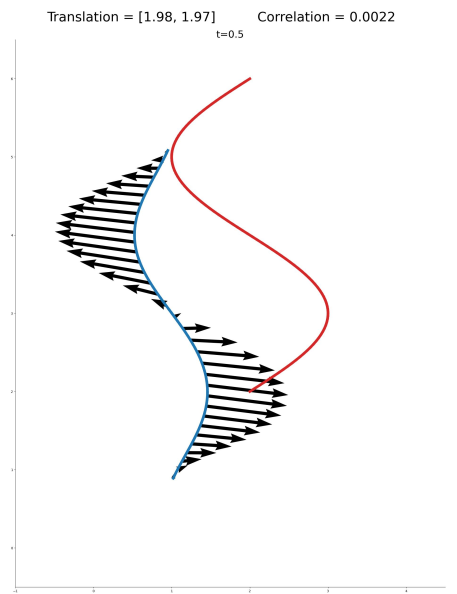

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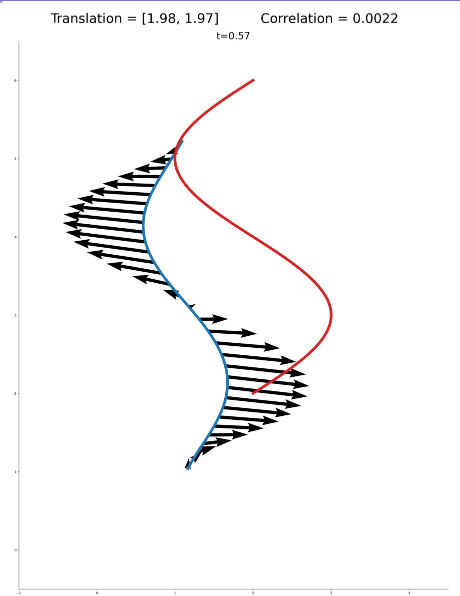

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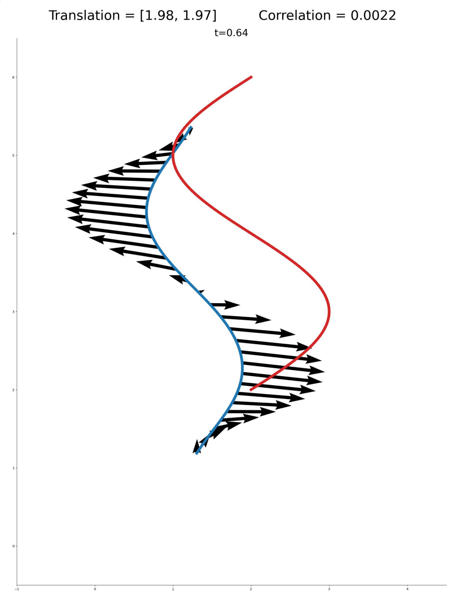

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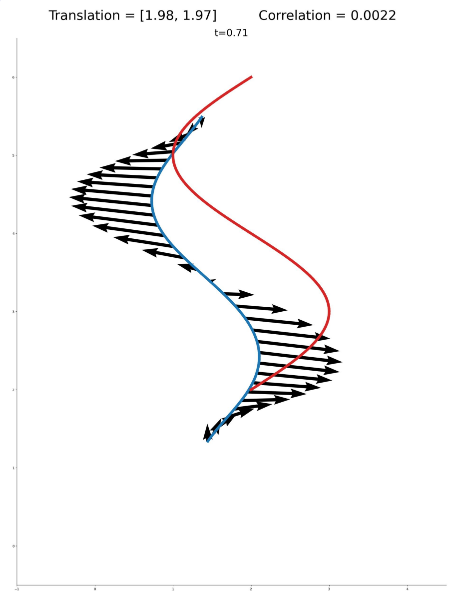

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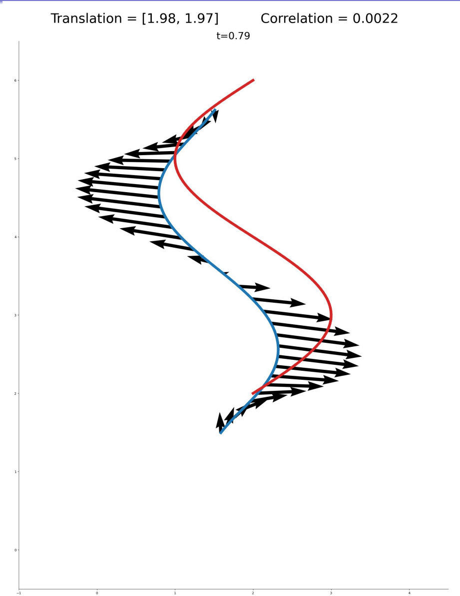

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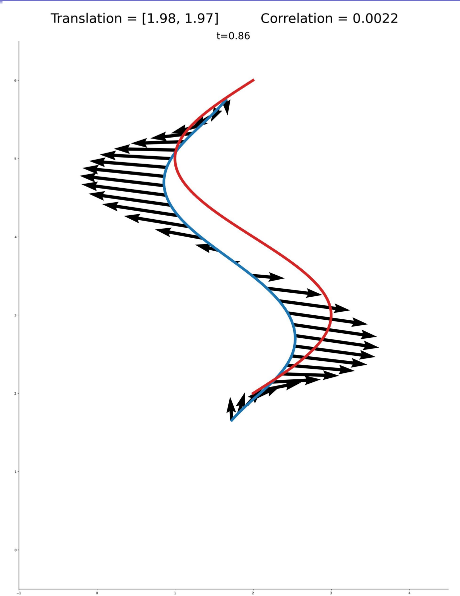

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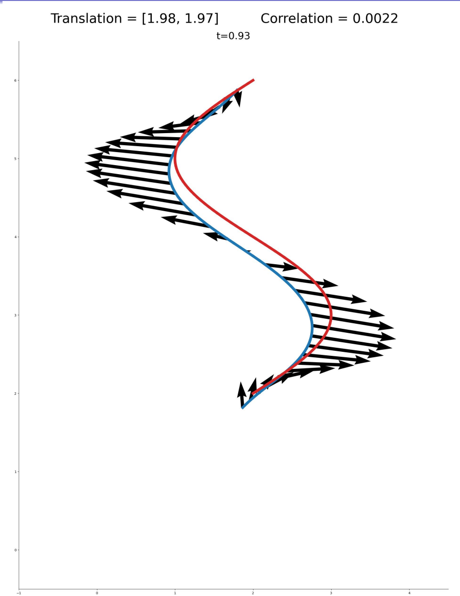

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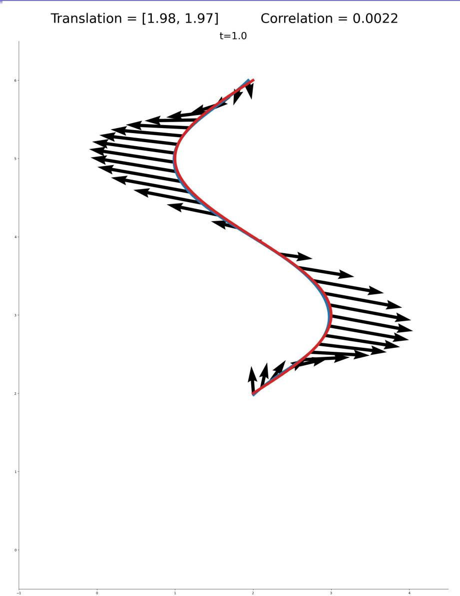

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